Economics 200B — Prof. R. Starr
University of California, San Diego — Winter 2010

Problem Set 3

(Adapted from an old final exam)

It's OK to work together on problem sets.

Questions 1, 2, and 3 are based on the following model of public goods and the solution concepts of competitive equilibrium, Lindahl equilibrium: Consider an economy of ten (10) identical households $i \in H$, a finite set of firms F, and two commodities known as x and g. Each household i, is endowed with (strictly positive) $\bar{X}^i$ of good x. Assume $\bar{X}^i > 1$. Good g is produced by firms $j \in F$ (all of which have the same constant returns technology), at the rate of one unit of output g for each unit of input x. g^i denotes household i's purchase of good g.

We define $G = \sum_{i \in H} g^i$. (1)

Let each i have a continuous weakly concave utility function

$u^i(x^i, G) \equiv x^i + (0.25)\min[G, 5]$, for x^i and $G \geq 0$.

That is, household i enjoys G up to a maximum of 5 units and likes each unit of G one fourth as much as he likes x. g^i and G are public goods. The utility function is continuous everywhere, but it is not differentiable with respect to G in the neighborhood of G=5. We contrast two solution concepts below: competitive equilibrium, Lindahl equilibrium.

<u>Competitive Equilibrium</u>

Assume marginal cost pricing: the price of x equals the price of g and we can set these prices at unity, $p_x = 1 = p_g$, for convenience. All firms run zero profits so household income is merely the value of endowment. We maintain the convention that households sell all of endowment and repurchase the amount they wish to consume.

Household i's budget constraint in a marginal cost pricing equilibrium reads

$x^i + g^i = \bar{X}^i$ (2)

where x^i is i's purchase of good x, and g^i is i's purchase of good g (good x acts as numeraire).

Household i's competitive market consumption choice problem is to

Choose x^i, g^i, to maximize $u^i(x^i, g^i + \sum_{h \neq i} g^h)$ subject to (2) (3).

In solving (3), household i treats the prices of x and g parametrically and treats the choices of g^h of other households, $h \neq i$, parametrically as well.

We define a competitive equilibrium for this economy as choices x^{*i}, g^{*i}, $G^* = \sum_{h \in H} g^{*h}$, fulfilling (2) and (3) for each household i so that all markets clear, that is, so that

$$G^* + \sum_{h \in H} x^{*h} = \sum_{h \in H} \bar{X}^h \quad (4).$$

Lindahl Equilibrium

We define the Lindahl budget constraint of household i as

$$x^i + q^i G^i = \bar{X}^i \quad (5)$$

where q^i is i's (personal) Lindahl price of the public good. Household i's Lindahl consumption choice problem is to

Choose x^i, G^i to maximize $u^i(x^i, G^i)$ subject to (5) (6)

where i treats q^i parametrically. We define a Lindahl equilibrium of the economy as an array of choices x^{*i}, G^{*i}, prices q^i, $i \in H$, fulfilling (6), so that $\sum_{i \in H} q^i = 1$, so that all G^{*i} (=G^*), $i \in H$, are equal and markets clear, that is, (4) is fulfilled.

1. There is a competitive equilibrium in this problem.
(i) Find competitive equilibrium prices and the resulting allocation. Explain.
(ii) The First Fundamental Theorem of Welfare Economics (Starr's *General Equilibrium Theory: An Introduction* , Theorem 12.1) says that the competitive equilibrium allocation is Pareto efficient. Is that true in this example? Why or why not?

2. Find Lindahl equilibrium prices and the corresponding Lindahl equilibrium allocation. This is probably simplest if you choose a Lindahl equilibrium that treats all households equally. Is it Pareto efficient? Explain.

3. There is a free rider problem in this example. Describe it. How does it affect the allocations in problems 1 and 2?

4. (Adapted from Walter P. Heller) Consider a Robinson Crusoe economy producing two outputs on a river. The upstream firm produces commodity x and water pollution in proportion to the output of commodity x. The downstream firm produces y, and output of y is reduced by the pollution coming from production of x. There is a lower bound of 0 on the output of x and y. The production relations of these commodities are

$x = L_x$

$y = L_y - x$ when this expression > 0

0 otherwise.

where L_x , L_y is the amount of labor going to production of x and y respectively. L_x , $L_y \geq 0$. Labor is inelastically supplied and leisure is not valued. $L_x + L_y = 1000$. Note that the production possibility set is nonconvex.

Robinson's utility function is

$u(x, y) = 12x + 8y.$

His income is precisely sufficient to purchase all of the goods x and y produced.

(i) The downstream firm treats the volume of x upstream parametrically. A Pigouvian tax on good x, τ , that will correctly reflect the external effect will have the property $\tau = p_y$. Then the price paid by buyers of x is $p_x + \tau$ but the price received by sellers is p_x .

Show that the allocation

$x = 0, \; y = 1000$,

is a competitive equilibrium with taxation, where $p_x = p_y = 1 = \tau$. You may assume the wage rate is 1. This is a corner solution so the first order conditions may be fulfilled as an inequality.

(ii) The allocation x = 0, y = 1000, fulfills the first order conditions for a local maximum of utility subject to technology constraint and hence for a Pareto efficient allocation. That is, $MRT_{x,y} = 2 > 1.5 = MRS_{x,y}$; the inequality is appropriate at a corner solution. In a convex economy, the first order conditions would be sufficient for Pareto efficiency, but this economy is nonconvex. Show that the allocation is Pareto inefficient and find a Pareto efficient alternative. (Hint: It may help to diagram this problem).